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I In the above theorem, the linear transformations T_1 and T_2 are assumed to be invertible. This is not a restriction, since any linear transformation can be extended to an invertible linear transformation on a larger Hilbert space. In fact, the above theorem can be generalized to the case where T_1 and T_2 are not invertible. In this case, the inverse transformations T_1^{-1} and T_2^{-1} are not defined on the entire Hilbert space, but only on the range of T_1 and T_2 respectively. The composition $T_1^{-1}T_2^{-1}$ is then defined on the range of T_2 .

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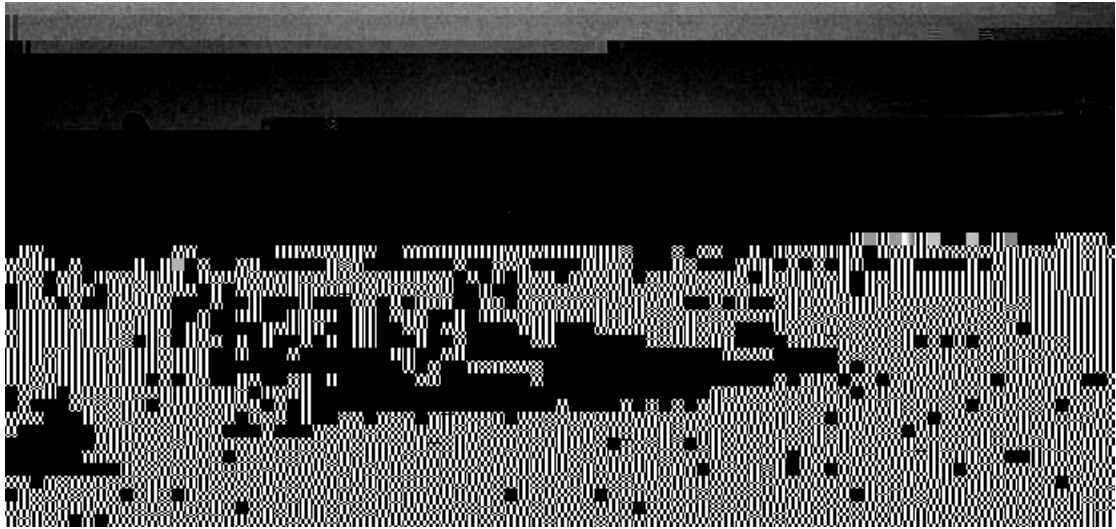


Fig. 1. The mobile laboratory at Resolution Island, Nunavut, Canada.

The mobile laboratory is a small, rectangular structure with a white exterior and a dark roof. It is situated on a flat, open area, possibly a tundra or a cleared site. The background shows a vast, flat landscape under a dark sky, suggesting a remote, high-latitude location. The image is heavily distorted with a dense, repeating pattern of vertical and horizontal lines, making the details of the laboratory and its surroundings almost completely illegible.

Let T be a function defined on \mathbb{R}^n by $T(x) = \begin{cases} 0 & \text{if } |x| \leq 1 \\ \infty & \text{if } |x| > 1 \end{cases}$. Then T is a convex function and T^* is a convex function. The function T is not continuous at $|x| = 1$. The function T^* is not continuous at $|x| = 1$. The function T is not differentiable at $|x| = 1$. The function T^* is not differentiable at $|x| = 1$.

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