DATABASES, RELATIONS AND FUNCTIONAL DEPENDENCES

by

Tim Merrett

(Tim Merritt obtained his B.Sc. in mathematics at Queen's in 1964 and did his doctorate in theoretical physics at Oxford. He then worked as an application programmer for IBM in Scotland and is now an associate professor in the School of Computer Sciences, McGill University.)

Given n sets, $D_1, D_2, \ldots, D_n$, (not necessarily distinct) [redacted]

$$R \subseteq D_1 \times D_2 \times \ldots D_n \quad .$$

Relations of this sort are of great interest to computer scientists specializing in databases. Databases typically store formatted data such as

[redacted]

[redacted]

GRADE = $\{A, B, C\}$.

Mathematicians have been familiar with binary relations (n=2) for a long time and the properties of important classes of binary relations (notably with both domains the same set) are well known: equivalence relations, partial orderings, etc. Not much seems to be known about relations of order greater than two, in spite of their growing practical importance.

One important concept of binary relations that has been extended to n-ary relations is that of functionality. A function is a special binary relation in which no more than one element of the second domain (traditionally called the range) may be paired with any element of the first domain (traditionally called the domain: we will not use traditional terminology in this discussion). We introduce the functional dependence $A \rightarrow B$ for domains A and B to assert that, in the context of a particular relation, there is a many-one relationship between the elements of A and those of B. A functional dependence, $X \rightarrow Y$, may also hold between sets, X and Y, of domains. In STUDENT RECORDS, for instance, there is the functional dependence

$$\{\text{STUDENT, COURSE}\} \rightarrow \text{GRADE}$$

This may be interpreted as saying that if we know the [illegible]

If we know that some functional dependences hold for a relation, can we derive additional dependences?

The following rules may be shown, from the definition, [redacted]

REFLEXIVITY	$X \supseteq Y$ implies $X \rightarrow Y$	
UNION	$X \rightarrow Y$ and $X \rightarrow Z$ implies $X \rightarrow Y \cup Z$	
DECOMPOSITION	$X \rightarrow Y \cup Z$ implies $X \rightarrow Y$ and $X \rightarrow Z$	
AUGMENTATION	$X \rightarrow Y$ implies $W \cup X \rightarrow Y$	
TRANSITIVITY	$X \rightarrow Y$ and $Y \rightarrow Z$ implies $X \rightarrow Z$	

where W, X, Y and Z are sets of domains. Either reflexivity union and transitivity or reflexivity, augmentation and trans- [redacted] itivity are axioms for functional dependences.

Now let us refine our definition of a relation to account for the fact that, in a database, it represents data physically stored on a medium such as a magnetic disc. It can be changed from time to time. A relation must now be defined as a set of subsets of the Cartesian product of its domains, together [redacted] ferences between accidental and essential functional dependences

It also shows how database workers consider relations to be time-varying in that they can be updated by addition of, deletion

on the number of n-tuples that may belong to the relation. If STUDENT RECORDS were unconstrained, it could have up to 27 n-tuples.

These arguments generalize immediately to the n-ary relation $R(D_1,D_2,\ldots,D_n)$. The quantities corresponding to 9 and 4 are, respectively, $\prod_i |D_i| / \min_i(|D_i|)$ and $n + 1$.

For numbers of n-tuples between these extremes we are faced with a large combinatorial problem. Even for STUDENT RECORDS there are 2^{27} possible instances to be examined in all or $\binom{27}{4} + \binom{27}{5} + \binom{27}{6} + \binom{27}{7} + \binom{27}{8} + \binom{27}{9}$ possible instances with from 4 to 9 n-tuples.

As an example of a complete enumeration we show the

The Department is presenting a

SYMPOSIUM ON STATISTICS

to honour Professor Emeritus George L. Edgett

on Monday, November 5, 1979

[illegible]

Professor Edgett, who joined the staff of Queen's [illegible]

[illegible]

[redacted]

Selwyn Caradus has been named Anglican chaplain at the university, effective this September.

He is an active member of St. George's Anglican Church, [redacted]

In August, he completed a master of theological studies from Queen's. After ordination, he plans to continue as a part-time teacher and part-time Anglican chaplain.

A specialist in a branch of mathematics called functional analysis, Caradus earned his first degree from the University of Auckland in his native New Zealand. His master's degree [redacted]

Jim Verner has presented papers recently at the Conference on Numerical Ordinary Differential Equations at Urbana, Illinois, in April, at the Department of Computer Science, University of Toronto in May, and at the Conference on Numerical Mathematics at Winnipeg in Steptember.

[redacted]

MASTER'S DEGREES AWARDED RECENTLY BY THE DEPARTMENT OF MATH AND STATISTICS

NAME	SUPERVISOR	TITLE
[redacted]		
[redacted]		
		Graphs and Digraphs
GUINAND, Paul Scott	D. Norman	Morse Theory on Banach Manifolds
[redacted]		

NEWS FROM GRADUATES

1968

Rick Bunt (Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598) writes:

After graduating from Queen's I entered the graduate program in Computer Science at the University of Toronto, from which I received my M.Sc. and Ph.D. degrees. In 1972, I joined the faculty of the Department of Computational [redacted]

[redacted]

1941

Harr Occomore (15 Chelford Crescent, Belleville, Ontario, K8N 4J8) writes.

After two years Faculty of Applied Science and Engineering University of Toronto, I spent 30 years in Outside Plant Design Engineering with Bell Canada. The last 12 years I was Supervisor Engineering Design at the Bell Canada Technical Training Centre, 11 Bay Bridge Road, Belleville, Ontario. Also during my "declining years" I had the pleasure of teaching calculus, statistics and probability and Business statistics at Loyalist College here in Belleville.

1973

Robert Preston (2514-2 Forest Laneway, Willowdale, Ont., M2N 5X7)

After graduating with an Honours B.Sc. degree in [redacted]

[redacted]

Among visitors to the Department who have arrived recently are the following:

1. Michael Bulmer - a theoretical biologist from Oxford working [redacted]

[redacted]

7. Gw n Evans - from Aberystwyth is the Visiting Senior Statistician with Statlab this term.

[redacted]

postdoctoral visitor in statistics who will be with us for the year.

9. Takashi A oh - is from the Science University of Tokyo, an

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[redacted]

The 28th Summer Research Institute of the American Mathematical Society will be held at Queen's from July 14 - August 1, 1980. It will be devoted to operator algebras and

[redacted]

LETTERS

We were pleased to receive so many letters in response [redacted] of news from old friends and graduates, and many suggestions for future issues.

Here are a few excerpts:

I have recently been preparing a senior academic high school [redacted]

I am currently the Head of the Mathematics Department at Fort William Collegiate in Thunder Bay. We are a small school (700 students) but have produced some very talented math graduates.

I enjoyed Ron Horn's article and would appreciate more articles in this vane. Math applications are very important [redacted]

While pursuing my academic career I found that my 'Course J' background was very relevant and gave me an

[illegible]

CONFERENCE ON RECENT PROGRESS ON NUMBER THEORY

From July 2 to 20, 1979, the Queen's Mathematics and Statistics Department was host to an international conference

on Recent Progress in Number Theory. About 125 mathematicians were in attendance, including more than 50 from the U.S., 30 from France, 25 from Canada; and about 20 from other countries including Brazil,

There is a new largest known prime, found last October by two California high-school students who announced their discovery at the West Coast Number Theory Conference held last Christmas.

It is

$$2^{21701} - 1$$

[illegible] of 6533 decimal digits

ALAIN CONNES RECEIVES HONORARY DEGREE FROM UEEN'S

[redacted]

of Doctor of Laws (honoris course) was awarded to the French

[redacted]

by A. J. Coleman and E.J. Woods.

Alain Connes was born and raised in the south of France, received his university education in Paris, and now holds a position at the University of Paris. At the age of 28, he became world-famous by solving in a very beautiful and constructive manner

[redacted]

PROBLEM SECTION

by

Peter Taylor

The following problem was posed in the last issue.

Problem No. 1

Suppose we have p standard coins, all identical except that one has a different weight from the others. We wish to use a simple balance to find the non-standard coin. Assume we have an inexhaustible supply of standard coins which we can use in the weighing.

Let $Q(n)$ be the maximum value of p for which the problem can be solved in n weighings.

To get a feeling for the problem discover that

$$Q(1) = 2$$
$$Q(2) = 5$$
$$Q(3) = 14 .$$

Find $Q(n)$ and prove that it holds.

Can you think of any other, more difficult versions of this problem?

Solution (by P.J. Cahen, Tunisia)

Consider first the simpler problem in which we know the odd

$$Q(n) = Q(n-1) + 3^{n-1} \; .$$

Comments.

Cahen's solution is neat and correct except that the underlined statement requires a bit of justification. The 3^{n-1} coins we use in our first weighing need not have been weighed against 3^{n-1} standard coins. They could have been divided up between the two

Problem No. 2

[illegible]

[illegible]

$$T(x_1,x_2,\ldots,x_n) = (|x_1-x_2|,|x_2-x_3|,\ldots,|x_{n-1}-x_n|,|x_n-x_1|)$$

Here is an example for $n = 5$.

$$\begin{aligned} x &= 4 \quad 7 \quad 8 \quad 1 \quad 4 \\ Tx &= 3 \quad 1 \quad 7 \quad 3 \quad 0 \\ T^2x &= 2 \quad 6 \quad 4 \quad 3 \quad 3 \\ T^3x &= 4 \quad 2 \quad 1 \quad 0 \quad 1 \quad . \end{aligned}$$

[illegible]

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[illegible]

planning quite a field trip this year. Not content with

[illegible]