

# Circuit Modeling of Carrier–Photon Dynamics in Composite-Resonator Vertical-Cavity Lasers

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**Abstract** This paper presents a circuit model for the carrier–photon dynamics in composite-resonator vertical-cavity lasers (CR-VCLs). The model is based on the rate equations for the carrier and photon populations in the different regions of the laser. The carrier and photon populations are represented by nodes in a circuit, and the transitions between these nodes are represented by branches. The circuit model is used to analyze the carrier–photon dynamics in CR-VCLs with different resonator configurations. The model is shown to be able to predict the carrier and photon populations in the different regions of the laser, as well as the carrier–photon dynamics in the different regions of the laser. The model is also used to analyze the carrier–photon dynamics in CR-VCLs with different resonator configurations. The model is shown to be able to predict the carrier and photon populations in the different regions of the laser, as well as the carrier–photon dynamics in the different regions of the laser.



recombination,  $v_{gk}$  is the group velocity of the optical mode in the lasing medium,  $\alpha_k$  is the percentage of the optical standing wave overlapping with the respective cavity,  $\tau_p$  is the photon lifetime,  $\tau_k$

where

$$I_k^{T1} = I_k^{D1} + I_k^{C1} \quad (19)$$

$$I_k^{T2} = I_k^{D2} + I_k^{C2} \quad (20)$$

$$I_k^{D1} = \frac{q N_{k0} V_{actk}}{2\eta_{ik} k} \left[ \exp\left(\frac{q V_k}{nkT}\right) - 1 \right] \quad (21)$$

$$I_k^{D2} = \frac{q N_{k0} V_{actk}}{2\eta_{ik} k} \left[ \exp\left(\frac{q V_k}{nkT}\right) - 1 + \frac{2q k}{nkT} \exp\left(\frac{q V_k}{nkT}\right) \frac{dV_k}{dt} \right] \quad (22)$$

$$I_k^{C1} = I_k^{C2} = \frac{q N_{k0} V_{actk}}{2\eta_{ik} k} \quad (23)$$

$$B_k^N = \frac{\lambda_p q g_k k}{\eta_{ik} \eta_c h c} \frac{g (k I_k^{T1})}{k (V_m + \delta^2)} V_m + \delta^2 \times \left[ 1 + \left( \frac{V_{act1}}{V_{act2}} \right)^{-1 k} \right]^{-1} \quad (24)$$

with

$$k = \frac{2\eta_{ik} k}{q V_{actk}} \quad \text{and} \quad N_k = k I_k^{T1} \quad (25)$$

Similarly, to model the photon dynamics  $dS/dt$ , we substitute the transformations (15) and (16) and the output power (8), into the rate equations (3). After applying appropriate manipulations, we obtain

$$2 V_m + \delta \frac{dV_m}{dt} = -\frac{V_m + \delta^2}{p} + \frac{\beta_1 N_1}{1} + \frac{\beta_2 N_2}{2} + \left\{ 1 g_1 \frac{g N_1}{1 (V_m + \delta^2)} + 2 g_2 \frac{g N_2}{2 (V_m + \delta^2)} \right\} V_m + \delta^2 \quad (26)$$

With some additional rearrangements and the definition of suitable circuit elements, (26) can be rewritten as

$$C_{ph} \frac{dV_m}{dt}$$

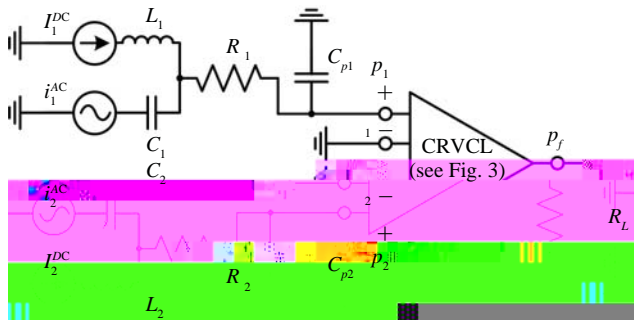


Fig. 4. Circuit setup to simulate the CRVCL equivalent circuit model.

TABLE I  
CRVCL DEVICE PARAMETERS USED IN EQUIVALENT  
CIRCUIT MODEL [5], [8], [15], [17]

Symbol	Parameter Name	Value
$\eta_1 \eta_2$	Current-injection efficiency	0.86
$\lambda$	Lasing wavelength	850 nm
$V_{act1}$	Top cavity active region volume	$1.2 \times 10^{-18} \text{ m}^3$
$V_{act2}$	Bottom cavity active region volume	$2.4 \times 10^{-18} \text{ m}^3$
$\Gamma_1 \Gamma_2$	Optical confinement factor	0.055
$g_1 g_2$	Lasing medium group velocity	$3.35 \times 10^{10} \text{ cm s}^{-1}$
$\Delta_1 \Delta_2$	Optical standing wave overlap factor	0.5
$\tau_1 \tau_2$	Carrier lifetime	2.6 ns
$p$	Photon lifetime	2.5 ps
$g_{10} g_{20}$		10



$$\left| \frac{s}{j_1} \right| = \left| \frac{1 \ 1g'_1 S_0 \ qd_1}{2 + i \ 1 \ 1 + 1g'_1 \Delta_1 S_0 - \frac{2}{1} \ 1 \Delta_1 g_{10} g'_1 S_0 - \frac{2}{2} \ 2 \Delta_2 g_{20} g'_2 S_0 \ \gamma} \right| \quad (32)$$

$$\gamma = \frac{i \ - \ 1 \ 1 + 1g'_1 \Delta_1 S_0}{i \ - \ 1 \ 2 + 2g'_2 \Delta_2 S_0} \quad (33)$$

$$\left| \frac{s}{j_2} \right| = \left| \frac{2 \ 2g'_2 S_0 \ qd_2}{2 + i \ 1 \ 2 + 2g'_2 \Delta_2 S_0 - \frac{2}{2} \ 2 \Delta_2 g_{20} g'_2 S_0 - \frac{2}{1} \ 1 \Delta_1 g_{10} g'_1 S_0 \ \gamma} \right| \quad (34)$$

above the RO frequency including the RC bandwidth, they do not compare well for frequencies below the RO frequency. This is not the case with the proposed circuit model where the results are comparable across all frequencies. It should be noted that the analytical model solves the rate equations algebraically obtaining a closed-form expression, whereas the circuit model solves the rate equations numerically by accounting for the interdependence between laser parameters. The proposed circuit model provides an improvement over the existing analytical model.

2) *Cavity Detuning Characteristics*: Fig. 7(a) shows the simulated CRVCL circuit model modulation response as a function of the percentage of the optical longitudinal mode overlapping with the top cavity ( $\Delta_1$ ), when only the top cavity is under direct modulation. The simulations depicted in Fig. 7(a) only vary ( $\Delta_1$ ) and assume the other cavity parameters are the same for both cavities. Although different optical field distribution may lead to changes in other cavity parameters, such as the gain coefficient and differential gain, the proposed equivalent CRVCL circuit model use these simplified assumptions. It can be observed from Fig. 7(a) that the modulation response has an increasing RO peak and greater modulation bandwidth as more optical field is confined in the top cavity. On the other hand, Fig. 7(b) shows the CRVCL modulation response as a function of  $\Delta_1$ , when only the bottom cavity is under direct modulation. Similarly, the modulation response can be engineered by varying the detuning between both cavities. However, it is interesting to observe that the modulation response exhibits the opposite trend as compared to Fig. 7(a).

The case when  $\Delta_1 = 50\%$  in Figs. 7(a) and 7(b) correspond to the same modulation response, which is also equivalent to the modulation response of a conventional VCSEL with the same photon density. When more optical field is confined in the top cavity, the modulation response becomes more damped (decreasing RO peak) and the modulation bandwidth decreases. Therefore, for a given  $\Delta_1$ , we can obtain two different modulation responses, depending on which cavity we have chosen to apply direct modulation. For an appropriate  $\Delta_1$ , the CRVCL would achieve a larger bandwidth than that of a conventional VCSEL with the same photon density. This is

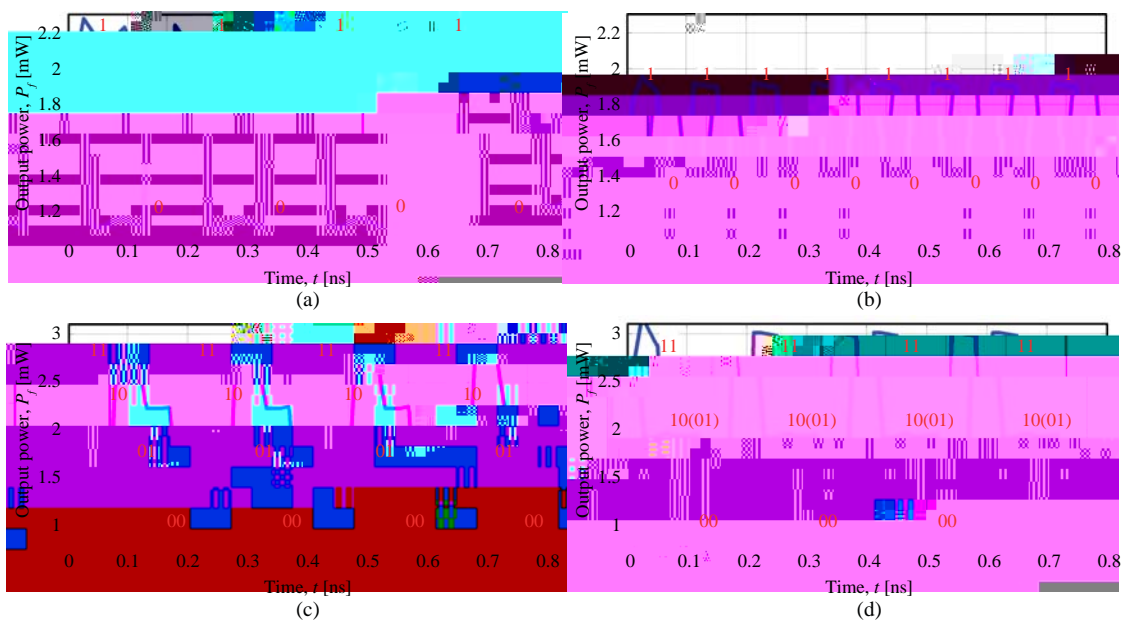


Fig. 8. Circuit modeling of optical output signal when only the (a) top cavity and (b) bottom



mode, another photon equation needs to be added and the electrical carrier equations (1) and (2) need to be modified so that the carriers are consumed by two simulated emission processes; we also need to consider the cross terms such as the cross gain compression and phase detuning of two longitudinal modes. Due to the additional interdependence between photons and carriers interactions, the rate equations can be only solved using numerical methods such as the Runge-Kutta method [15]. In this paper, we show that by mapping the CRCVL rate equations into a circuit model, SPICE analysis can be used as an efficient and accurate implementation of numerical calculations. And we believe that this circuit approach would also provide a smooth upgrade when more complex higher-order CRVCL effects are incorporated in the future. Nevertheless, the current circuit model will still be useful when a predominant single longitudinal mode is desired for applications such as push-pull modulation [8]. This single longitudinal mode condition can be achieved by setting a proper cavity detuning and/or gain/cavity spectral alignment.

In addition, we have verified that using the proposed circuit model, the asymmetries between the coupled cavities can be used to engineer the CRCVL modulation response. We also confirm the CRVCL's uni-

