A Continuum Approach to the Distribution of Plasma in a Pulsar Magnetosphere

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Introduction

section 4, various possibilities in modelling the magnetic eld are considered. Section 5 describes the computational methods used to generate data sets and plots. Finally, in section 6.4 we discuss our density distributions in the context of the eclipse light curve and the reasonably successful model due to Lyutikov and Thompson (2005).

2
PSR J0737-3039

PSR J0737-0309 is the rst known double pulsar binary system, and aside from being a laboratory for precise tests of General Relativity it has provided a unique opportunity to study the absorption of radio frequency radiation from one pulsar in the magnetosphere of the other.

2.1 Basic Parameters

Pulsar B's radius is taken to be 10 km, and its mass is about 1.25 solar masses. It orbits with A around their center of mass with a semimajor axis of 8.8 10^8 m a eccentricity 0.088, with a period of 2.45 hours. A and B's spin periods are 23 ms and 2.8 s respectively, and both are visible in the radio spectrum[1]. B's dipole moment is estimated as approximately 3.5 10^{26} J/T.

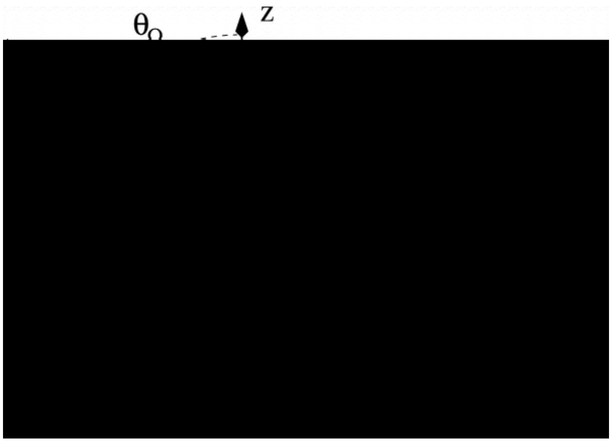

Figure 2.1: Geometry of the eclipse, with origin at pulsar B and z-axis normal to the orbital plane. The spin axis $_B$ makes an angle with the z-axis, and the magnetic dipole moment $_B$ makes at an angle to $_B$. (Figure from Lyutikov and Thompson (2005))

2.2 Eclipse Light Curve

The orbit is nearly (but not quite) edge on as viewed from Earth, so that once per orbit pulses from A pass through B's magnetosphere with some impact parameter z_0. The intensity of radiation recieved from A is modulated in time over the 30s eclipse, showing largely opaque periods as well as nearly transparent windows at the rst and second harmonics of pulsar B's spin frequency (see g. 2.2). These features are explained fairly well by Lyutikov and Thompson (2005) using a simple geometric model of pulsar B's magnetosphere as a rigidly rotating oblique

[1] Breton et al. (2008)

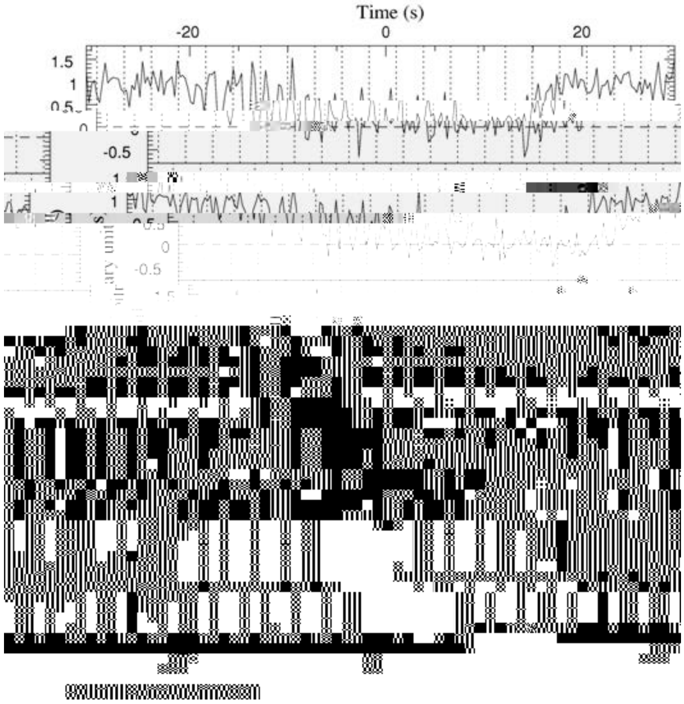

Figure 2.2: Light intensity curves from three eclipses of J0737-0309 A by B are shown; the bottom panel is the average of the three. The dotted vertical lines represent the timing of pulses from B. (McLaughlin et al., 2004)

magnetic dipole, with constant plasma density along those eld lines with maximum extension between some inner and outer radius $R < R_{max} < R_+$, measured at the magnetic equator. Field lines closing inside or outside this radius are given zero density. These assumptions are contrary to those of this thesis, as our method gives varying density along eld lines. Their model is successful enough, however, that a detailed study was carried out using our method with their best t parameters wherever possible. The implications of our method in the context of Lyutikov and Thompson (2005) are discussed in section 6.4. They provide best t values for the parameters R , R_+, the angles , and , and the impact parameter z_0 (see Fig. 2.1). Of primary interest to this thesis are R_+ and , since the computation described in section 5 calculates a density pro le but does not go as far as to simulate the eclipse. R_+ can be used to limit the computation to a region comparable to the size of the magnetosphere (i.e. the half-width of the eclipsing material, which is determined from the duration of the eclipse and the orbital radius and period to be $1{:}5 \quad 10^7$ m). One needs such a limit, as the assumption of corotation breaks down at the light cylinder. Additionally, under the strict assumption of corotation the density calculation diverges for eld lines approaching the light cylinder, and the maximum density attained in a calculation depends strongly on the limit R_+. Lyutikov and Thompson's best t value for the angle between the dipole moment and the rotation axis is 75 , and this is taken as a starting point for our investigations.

3

The Magnetosphere at Equilibrium

The computational tool making this study of the magnetosphere possible is an integral of the motion due to Henriksen and Rayburn (1973); some key points of the derivation are summarized here. The notation primarily follows that of Fock (1964), taking the (+ { { {) convention for the Minkowski metric.

We consider the region surrounding a pulsar isolated from external gravitational influences (in the context of PSR J0737-3039, we neglect the effect of pulsar A's gravity on B's magnetosphere). The surrounding matter is treated as a single fluid plasma with stress-energy-momentum tensor (see Fock (1964))

$$T_M = (\quad + p/c^2) u \; u \qquad pg \; : \tag{3.1}$$

where p is the pressure, is the total mass density in the local inertial frame (including the mass equivalent of the plasma's compressional potential energy), and u is the four velocity.

The metric g is taken to be the Schwarzschild metric, with line element

$$ds^2 = g \; dx \; dx \;=\; 1 \quad \frac{2GM}{rc^2} \quad c^2 dt^2 \quad \frac{1}{1 \quad \frac{2GM}{rc^2}} dr^2 \quad r^2 (d \; ^2 + \sin^2 \; d \; ^2) : \tag{3.2}$$

Defining

$$1 \quad \frac{2GM}{rc^2} ; \tag{3.3}$$

the four velocity is

$$u \;=\; \quad p \!= \; \frac{dx}{dt}$$

$$= \; \frac{1}{\sqrt{1 \quad \frac{r^2 / \quad + r^2 \, \dot{}^2 + r^2 \sin^2 \quad \dot{}^2}{c^2}}} : \tag{3.4}$$

Under the assumption that the plasma corotates with the pulsar, simplifies to

$$= \; \frac{1}{\sqrt{1 \quad \frac{r^2 \sin^2 \quad ^2}{c^2}}} : \tag{3.5}$$

This implies a light cylinder radius of $R_{lc} = \quad \frac{c}{}$, which is close to the value $\frac{c}{}$ seen throughout the literature since $\quad (1 \quad 3 \quad 10 \; ^5)$ at this radius.

The use of the Schwarzschild metric in describing the geometry around a pulsar is justified under several assumptions:

- It is a vacuum solution, so is a good approximation only if the total gravitational effect of the magnetosphere is negligible compared to that of the pulsar. We believe this to be the case, since the plasma is many orders of magnitude less dense than a neutron star interior.

- It assumes a spherically symmetric mass distribution as the source of gravitation. In reality the mass distribution is distorted due to centrifugal effects.

- It neglects the effects of the pulsar's rotation on the geometry, so we require that the pulsar's angular momentum J is not too large. A more complex treatment could be done using the Kerr metric, which reduces to the Schwarzschild metric in the limit that $\frac{J}{Mc^2r} \ll 1$. The angular momentum of a pulsar is difficult to predict since the equation of state is unknown, and thus the moment of inertia cannot be calculated. As a rough estimate, we idealize pulsar B as a uniform sphere (radius $\sim$ 10 km) with moment of inertia $I = \frac{2}{5}M_BR_B^2$ to find
$$\frac{J}{Mc^2r} \approx \frac{2}{5}\frac{R_B^2\,\Omega_B}{c} \approx 3 \times 10^{-5}\frac{R_B}{r} \tag{3.6}$$
which in our calculation, is at most 3×10^{-6}. In light of this, the gravitational effects of pulsar B's rotation should be negligible; however, it should be noted that the moment of inertia is unknown by a large factor, and the Kerr geometry will begin to become significant for faster-spinning pulsars.[1]

Under these assumptions, we proceed in using the Schwarzschild metric in the corotating frame with the understanding that the calculation is restricted to the region $R_{pulsar} < r < R_{lc}$. R_{lc} is the "light cylinder" beyond which the assumption of corotation would imply velocities greater than c. Due to the strong magnetic field, we suppose that charged particles are confined to magnetic field lines, executing helical motion. Field lines that close a reasonable distance within the light cylinder may then be capable of supporting an equilibrium density of plasma.

Adding the electromagnetic field tensor to the equation 3.1 gives the total mass-energy-momentum tensor $T^{\mu\nu}$. The equations of motion follow from the conservation laws given by setting the covariant derivative of $T^{\mu\nu}$ equal to zero.

$$\begin{aligned} T^{\mu\nu} &= T_M^{\mu\nu} + T_E^{\mu\nu} \\ T^{\mu\nu}{}_{;\nu} &= 0 \end{aligned} \tag{3.7}$$

[1] The double pulsar system may allow a unique opportunity to measure the moment of inertia of pulsar A

(4.1)

Here the polar angle refers to the axis of rotation, is the angle between the rotation axis and the e ective magnetic dipole moment, and $= \frac{r}{R_{lc}} \quad t$ (See Higgins (1996)).

The magnetic Deutsch eld reduces to a magnetic dipole for $R_{pulsar} \quad r \quad R_{lc}$. Considering that the eclipsing material of pulsar B has a half-width of 1.4 10^7 m [1] and the light cylinder is about 1.3 10^8 m, the Deutsch eld will begin to give distinct results in the outer magnetosphere.

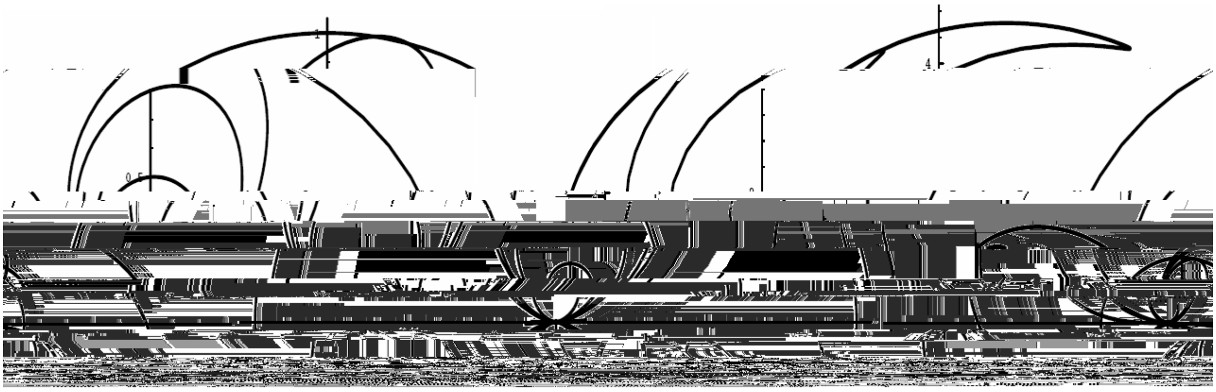

Figure 4.1: A selection of Deutsch magnetic eld lines that close within the light cylinder is shown at left, where the large circle represents the light cylinder. At right, a selection of open Deutsch magnetic eld lines is shown on a larger scale, where the small circle now represents the light cylinder. (gures from Henriksen and Higgins (1997))

Deutsch elds have been used in describing the exterior of a pulsar, for example by Henriksen and Higgins (1997), Quadir et al. (1980) and McDonald and Shearer (2009).

4.3 Plasma Fields

The Deutsch elds are vacuum solutions to Maxwell's Equations, and thus completely neglect the electromagnetic elds due to the ow of plasma in the magnetosphere. The plasma elds could in principle be added to the Deutsch elds by the principle of superposition; the problem is then to self-consistently solve for the charge and current densities and electromagnetic elds. This is a daunting task both analytically and computationally, and is beyond the scope of this paper. See McDonald and Shearer (2009) for a 3D computation that self-consistently nds equilibrium charge distributions using a superposition of Deutsch elds and plasma elds.

[1] Lyutikov and Thompson (2005)

5
Numerical Density Calculation

A major analytical difficulty is the loss of symmetry when the angle between the dipole moment and the axis of rotation is nonzero, which seems to be necessary in describing a real pulsar. For

5. NUMERICAL DENSITY CALCULATION

The reference points should be chosen with the limiting radius in mind, since the eld lines

Results and Discussion

6.1 Dependence on

Without knowledge of the equation of state of the plasma, we must choose a value of in the relation p / . Three choices of between 1 and 2, and the corresponding density distributions are shown in Figure 6.1.

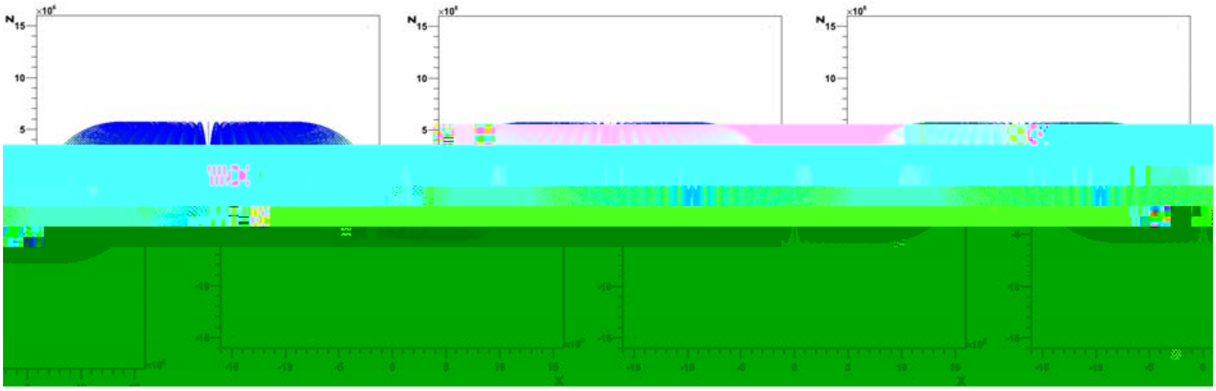

Figure 6.1: The axisymmetric case is shown for three values of : = 1:

Figure 6.2: The axisymmetric case, with the magnetic dipole calculation at left and the calculation using Deutsch elds at right, for eld lines that close within 1.5 10^7m.

Figure 6.3 shows the dipole eld (left) and the Deutsch eld in the extreme case in which the dipole moment is orthogonal to the rotation axis (= =2). Field lines closing within $0{:}5R_{lc}$ are plotted. The distortion in the dipolar structure of the Deutsch elds can clearly be seen for eldlines extending along the z-axis, while the eld lines near the rotational equator $z = 0$ do not extend as far radially and more or less maintain their dipolar shape. Note that the upper right panel of gure 6.3 shows the Deutsch magnetosphere to be narrower horizontally than the dipole elds; this is due to the Deutsch elds beginning to twist at large radii, as seen by comparing the bottom two panels. The viewing angle in the top right panel does not necessarily show the Deutsch magnetosphere at its maximum width, as is the case in the top left panel.

6.3 Comparison with McDonald and Shearer (2009)

McDonald and Shearer (2009) use a superposition of Deutsch elds and plasma elds in a 3D simulation to nd stable charge distributions by self-consistently moving charges around until a stabilty condition is met. It is important to recognize the distinction between this type of simulation and the one carried out in this thesis: McDonald and Shearer (2009) nd distributions of non-neutral plasma in which the electromagnetic forces are balanced, while our calculation nds mass distributions when the thermal, gravitational and centrifugal e ects are balanced (under the assumption that the electrostatic e ects of the charge separation are negligible). A full simulation including these e ects in a plasma density calculation would be an ambitious, but potentially worthwhile endeavour.

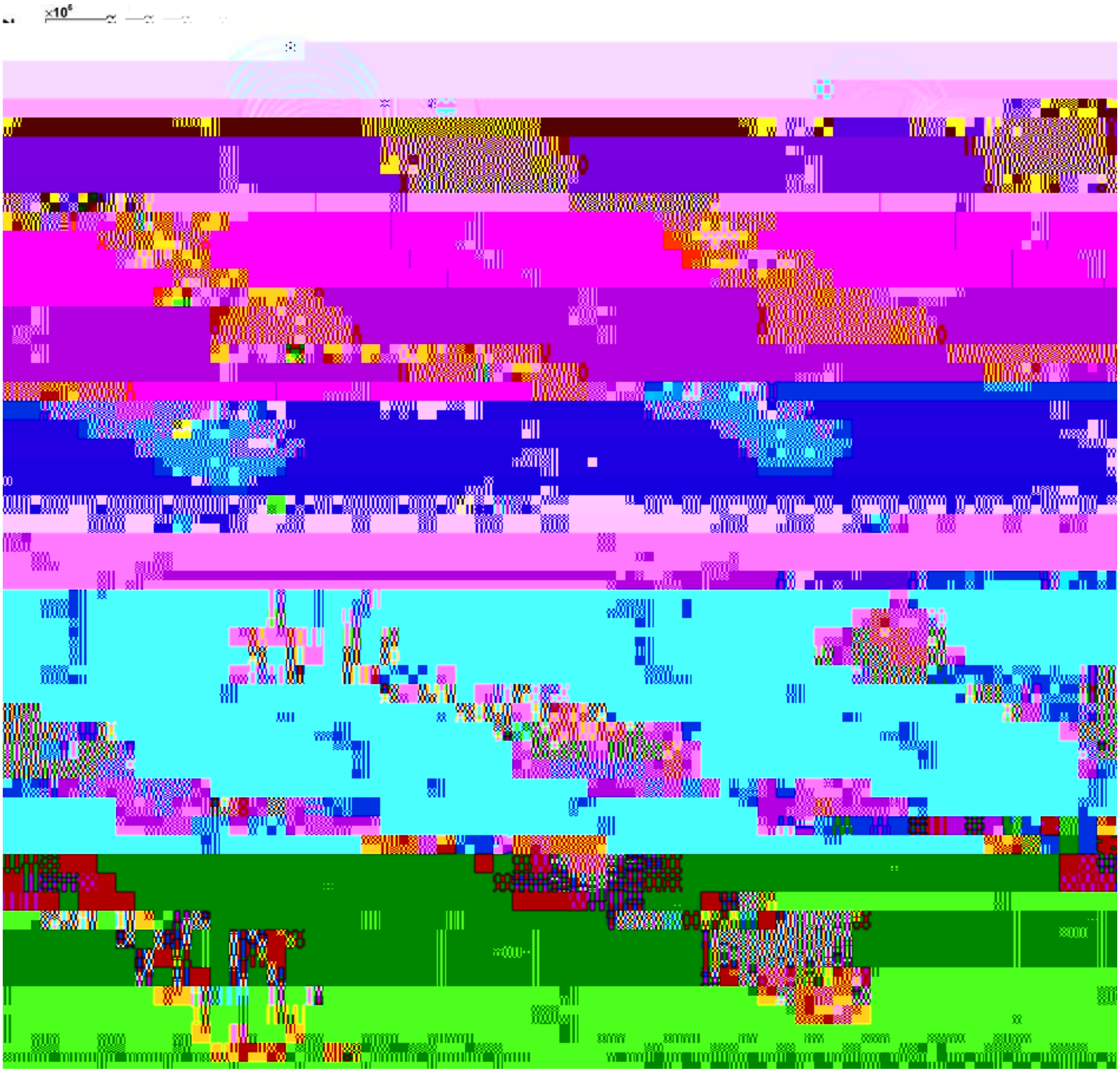

Figure 6.3: The orthogonal case, restricting field lines to one half of the light cylinder radius, $\sim 6.7 \times 10^7$m. The left figures shows the dipole fields and the right figures shows the Deutsch fields; the bottom figures are the same calculations rotated by 90° so that the dipole moment is out of the page. Using $\alpha = 1.8$, the dipole calculation had a relative maximum and minimum density of 250 and 1×10^{-4}, while the maximum and minimum for the Deutsch field calculation were 245 and 1×10^{-3} respectively.

6.4 Comparison with Lyutikov and Thompson (2005)

Lyutikov and Thompson model the density in the magnetosphere as being concentrated uniformly on a set of field lines with maximum extension within a fairly narrow range $R_- < R_{max} < R_+$. The assumption of constant density along field lines is contrary to our approach. Further, their criterion for the maximum radius is based on estimates of the "size" of the magnetosphere, and so they measure this radius with respect to the magnetic equator; as a result the limiting fieldlines, and thus the density distribution, are axially symmetric about the magnetic axis. Equations 3.9 and 3.10 suggest that the density distribution should be axially symmetric about the rotation axis, at least for initial points satisfying this symmetry; however when we restrict the calculation to field lines that close within a certain radius we obtain a density distribution that is not symmetric about either axis. When our calculation is restricted to a similar maximum radius we find relative densities ranging from 1 at the initial radius $r = 100$km down a minimum of about 1×10^{-4} (blue) before increasing to about 4.5 (red), when the parameter $= 1.8$. Lower values of   have the effect of increasing the maximum and decreasing the minimum. Given these considerations, we raise two issues with the model due to Lyutikov and Thompson (2005):

1. Their selection of which field lines to include uses a maximum distance measured along the magnetic equator, rather than along the equator of rotation. In our model we suppose that the ability of a field line to support an equilibrium plasma density depends on how closely it approaches to the light cylinder, and so the shape of the limiting field lines we include depends on the angle . In fitting the angle , Lyutikov and Thompson's magnetosphere stays the same shape.

2. Their model neglects to consider the equilibrium density along field lines, and assuming constant density seems to be a gross oversimplification.

As a possible explanation of their results, we note that the high density regions we calculate seem to lie approximately in a plane, which is not normal to the dipole moment or the axis of rotation, but is inclined somewhere between. If this was primarily the matter affecting the eclipse, then under Lyutikov and Thompson's assumptions the dipole moment is normal to this plane, which leads to an underestimation of the angle . One can then ask if there is angle , in our model, which gives a density distribution concentrated around a plane normal to Lyutikov and Thompson's best fit for the parameter , in the hopes that our density distribution can reproduce the eclipse light curve with a different value of . Figures 6.7 and 6.8 show calculations with the Deutsch fields for $= 80^\circ$ and $= 85^\circ$ respectively. As   approaches 90° the distribution becomes complicated, but we attempt to draw a line (really a plane extending into the page) around which the density distribution is roughly centered.

This idea weakly suggests that the angle   is closer to 90° than predicted by Lyutikov and Thompson (2005), though a detailed study is clearly necessary since our density curves are drastically different than theirs.

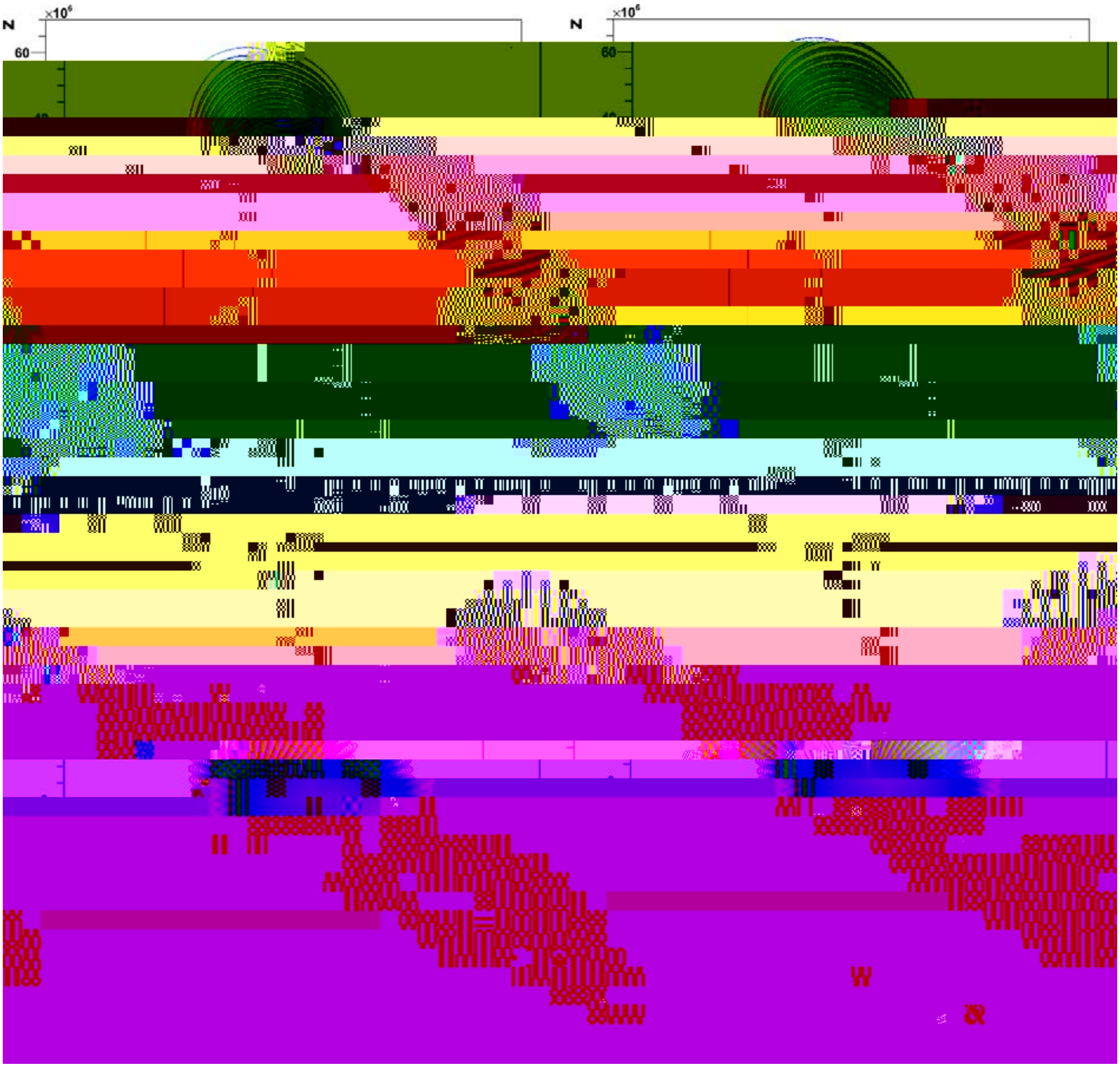

Figure 6.5: Using the same parameters as in 6.4, but restricting the field lines to those close within one quarter of the light cylinder radius, as measured at the rotational equator. Significant differences between the fields are still seen. The bottom frames show the same calculations rotated by 90°. In the dipole calculation the relative density reached a maximum of 36 and a minimum of 3.5×10^{-4}, and in the Deutsch field calculation it reached a maximum of 35.9 and a minimum of 7.7×10^{-4}.

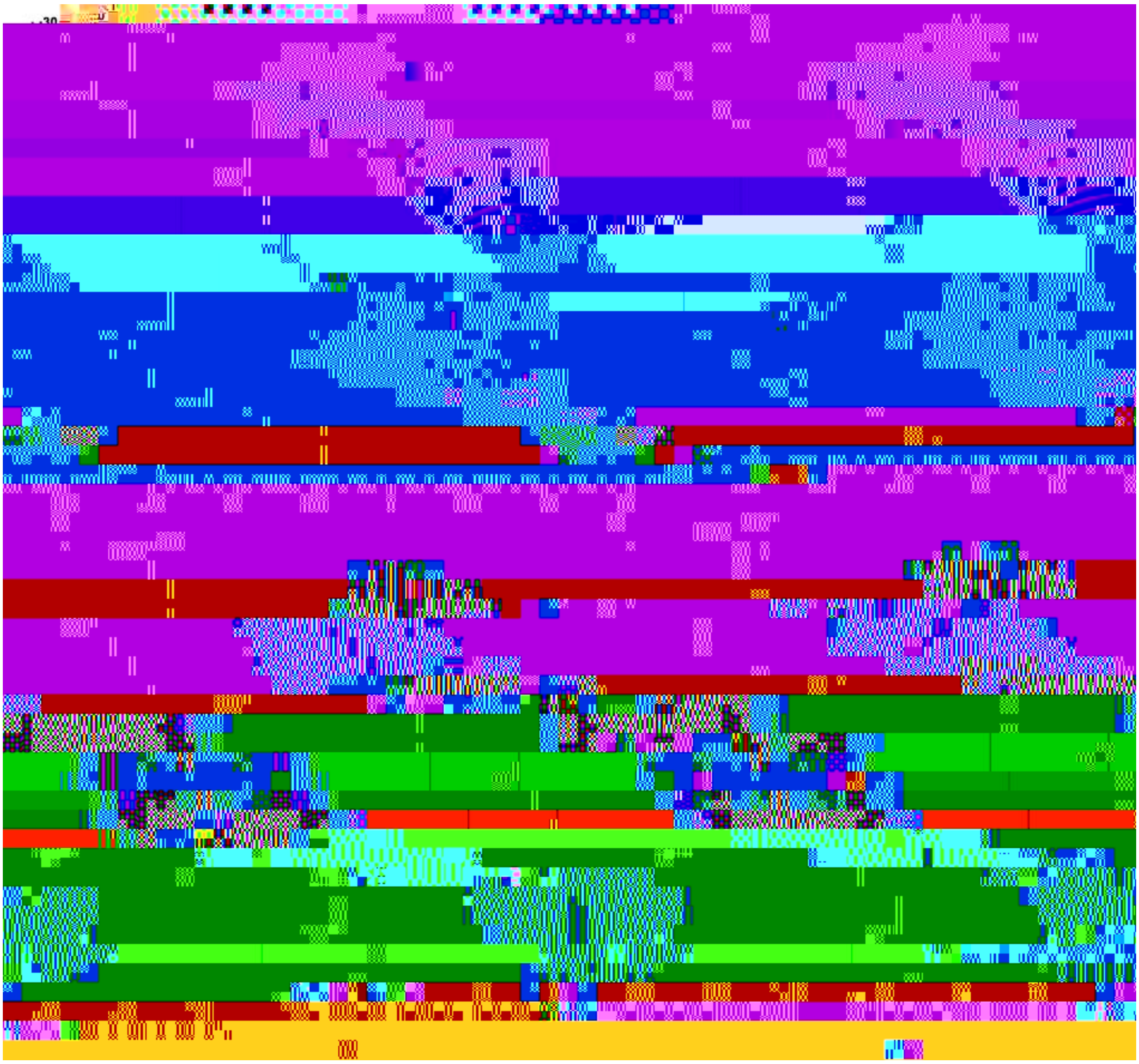

Figure 6.6: Once again using the same parameters as in 6.4, we now restrict the field lines to those close within 1.5×10^7m $\approx 0.11 R_{lc}$, as measured at the rotational equator. The fields appear similar at this scale. The bottom frames show the same calculations rotated by 90°. In the dipole calculation the relative density reached a maximum of 4.61 and a minimum of 9.5×10^{-4}, and in the Deutsch field calculation it reached a maximum of 4.63 and a minimum of 9.4×10^{-4}.

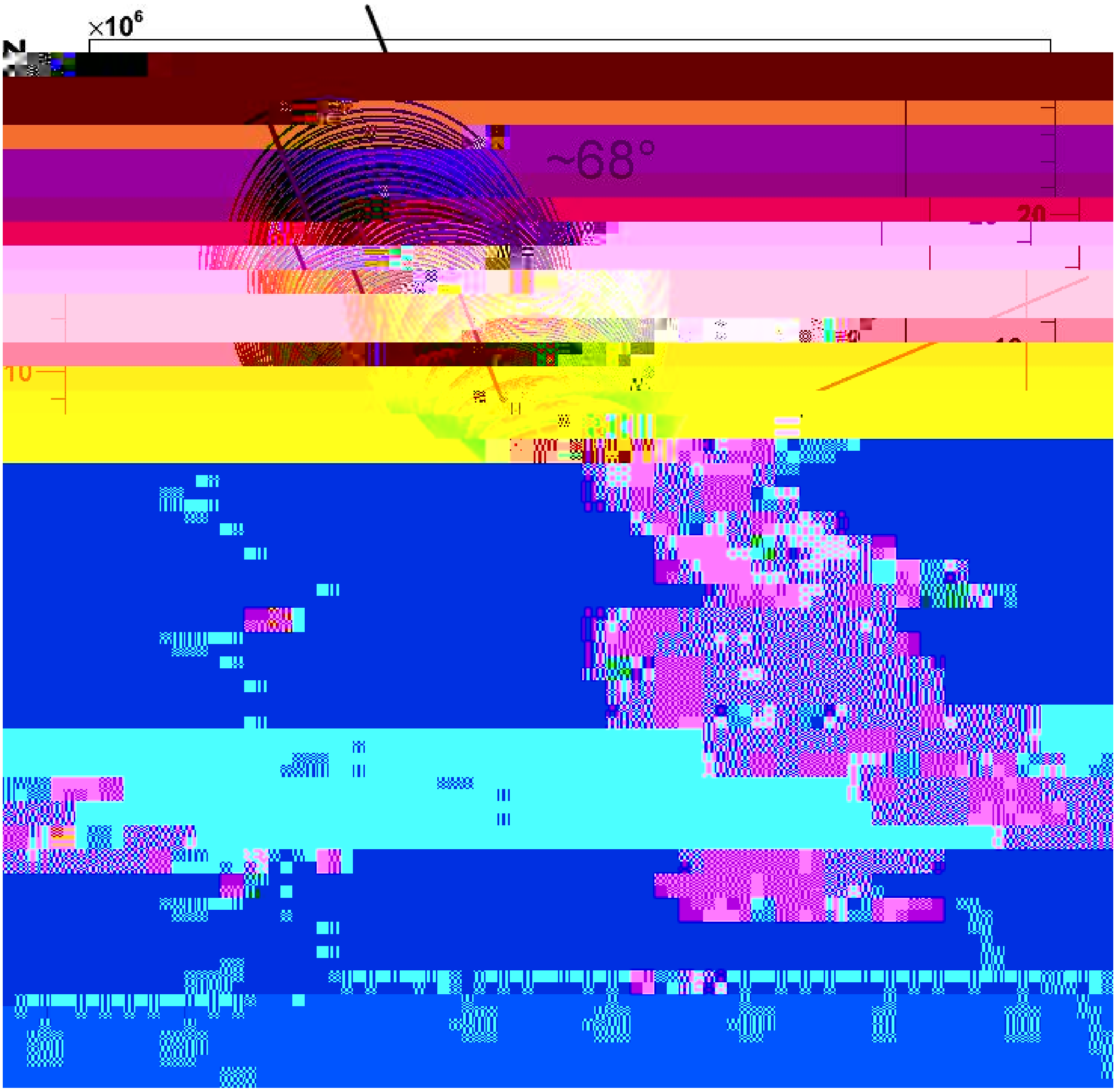

Figure 6.7: The Deutsch eld lines closing within 1.5 10^7m are shown for = 80 . A line is drawn representing a plane about which the density is roughly centered. A perpendicular line represents the normal to this plane, which Lyutikov and Thompson's analysis implicitly takes to coincide with the dipole moment. We nd this line to be at an angle of about 68 , though placement of the line is very subjective.

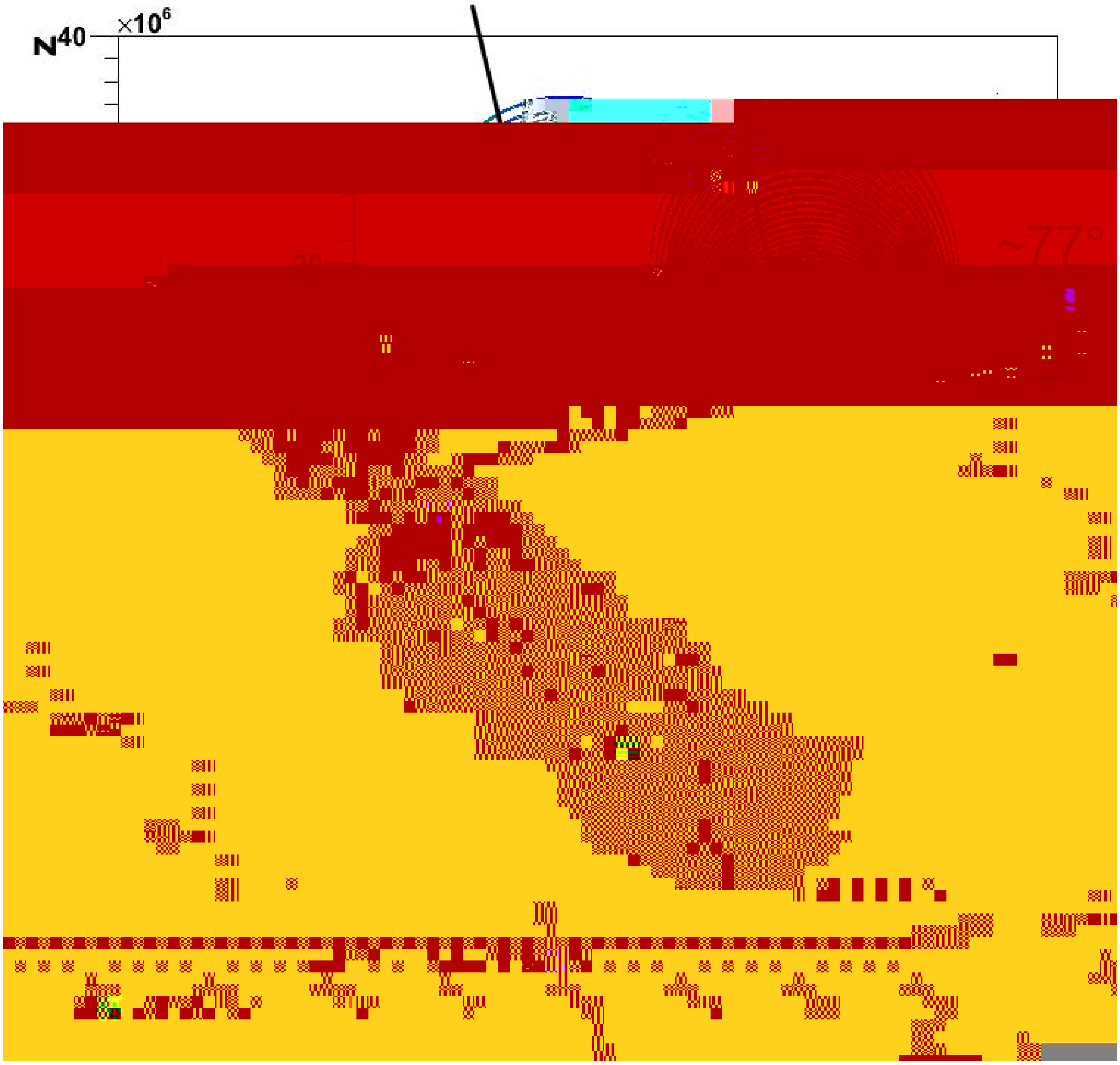

Figure 6.8: The Deutsch eld lines closing within 1.5 10^7m are shown for = 85 . A line is drawn representing a plane about which the density is roughly centered, but in this case the distribution is more complicated and the uncertainty in the slope is even larger. We give a value of about 77 for angle the normal to this plane makes with the rotation axis, though we recognize that the distribution takes on a new structure as approaches 90 .

7 Conclusions

Our approach is novel to that in the literature, and gives results inconsistent with the ideas of Lyutikov and Thompson (2005) in particular. The shape of the absorbing material, as well as the plasma density distribution within it, is seen to change with the angle . The density along eld lines closing within some radius decreases away from the pulsar until the e ects of corotation become signi cant and the density rises dramatically. It is clear that a full simulation of the eclipse light curve, along with an independent tting of parameters, is necessary in order to fully evaluate our model in the context of the pulsar B. Our results di er drastically from Lyutikov and Thompson's, so it is not clear whether or not a best t to our model would provide a slight modi cation of their parameters or a completely di erent set of parameters.

Within the radius $r = 1.5 \quad 10^{7}$